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ANTENNA ARRAY ELEMENT STUDY

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J. W. Marchetti, Inc.

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FOREWORD

This final report was prepared by M. R. Haslam and Z. M. CiscoeBrough of J. W. Marchetti, Incorporated, 12 Mercer Road, Natick, Massachusetts, under Contract AF30(602)-3707, project no. 4506, task no. 450604. Secondary report number is G149QR4. RADC project engineer was Edward J. Christopher (EMATA).

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ABSTRACT

An investigation and analytical study of antenna array elements has been carried out to determine the influence of an individual element of specific characteristics on the performance of an array and, on the basis of element performance, to present data which would assist the element designer in selecting the optimum element. A comprehensive literature survey forms parts of the basis for, and becomes a supplement to, the analytical study.

While the literature survey covers and records any type and size of array with any number of elements, the analytical investigation analyzes 49- to 215- element behavior and results in a determination of the 1000-element characteristics in the UHF (400 Mc) through X-Band (10.5 Gc) range of frequencies. Other parameters and limits for the analytical investigation include:

(1) Power handling capability (100 watts to 20 kilowatts)

(2) Terminal impedance (25 ohms to 1000 ohms)

(3) Signal bandwidths (from 10% at UHF to 5% at X-Band)

(4) Gain

(5) Polarization

(6) Beamwidth

For the literature search portion of the project (Phase I), a retrieval file system was designed and the file keysort cards for the system produced;

literature sources and contacts were established and are being used for data retrieval and literature source data.

Phase II, ananalysis of array elements and performance patterns.

Phase III, involves the final presentation of all findings.

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EVALUATION

1. Contract AF30(602)3707 entitled, "Array Element Study", is an engineering investigation to provide array element data in a tabulated card file form which would facilitate location of pertinent element data and which could easily be updated. A literature search and analysis of available element data has been performed. Results of the search were tabulated on notched cards, which make use of a simple numerical coding scheme that allows one to readily obtain desired element data by use of known performance requirements. For example, if a particular array gain is required then one can use the appropriate code number to select only those data cards on which that desired gain is specified. Knowledge of additional requirements, such as polarization, beamwidth, etc., further reduces the number of data cards to be checked, until only the most pertinent cards remain. In addition to specific element data, each of the cards contains the reference source from which the data was obtained along with an abstract of the article. The card file presently consists of approximately 500 data cards.

2. The analytical portion of this effort was directed toward further analyzing those areas in which available researched data was limited or for which major contradictions existed between various investigators. The analysis resulted in the development of a unique technique for predicting array pattern performance. The primary result of the analysis was in the definition of an optimum array element pattern, optimum from the standpoint of providing constant impedance VS scan characteristics, over a specified scan sector, when a number of such elements are used to constitute a phased array antenna. A synthesis based on the results of the analysis was initiated late in the study. The synthesis will provide a means by which various array functions, such as mutual coupling can be defined. The functions thus obtained will be used to provide element design parameters within the limits of the specific element considered. Further analytical effort resulted in the development of a technique termed "Pattern Ratio Techniques". It provides a means by which the impedance VS scan characteristics of an array can be calculated by using the ratio of the patterns obtained when a single element in the array is driven with three different impedances with all other array elements terminated. This technique offers significant simplicity over conventional means for measuring the impedance VS scan characteristics of an array.

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SECTION I

INTRODUCTION

Under Contract AF 30(602)-3707, J. W. Marchetti, Inc. has carried out an investigation and antenna array element study. Acquired information, was included in a preliminary study of element-array performance characteristics based upon this information. The study and analysis result, presented in a useful form with the retrieved information will aid the phased-array designer in selecting the optimum element on the basis of its performance in the array.

The final summary of the study and delineation of the three interrelated phase s are discussed in the following paragraphs.

hase I. Retrieval and recording of material - Array element retrieval and recording information in an expedient form for the use of the array designer and the study project personnel. This phase included the following tasks:

File card design and production (file system formation).

Establishment of literature sources and contacts,

Information retrieval from literature sources.

Recording literature to Keysort file cards.

Card Notching (for coded selection of data by designers).

Classification and grouping of elements,

Phase II. Analytical study and investigation - involved the following tasks:

a. Element types, patterns and array pattern study as described in the literature.

b. An extension of the literature study to include mathematical analysis of elements and antenna patterns.

c. Determination of an optimum element for a specific array performance and pattern.

d. Planning for optimum presentation of all information.

Phase III. Presentation of retrieved and analyzed information - involved tasks:

a. Oral presentation.

b. Technical Documentary Reporting.

c. Graphic presentation of analysis results (review type discourse).

d. Finalized card file and retrieval system.

SECTION II

PHASE TASKS COMPLETED

The final project effort has been directed toward completion of the Phase I and Phase II investigation and study. The final result is Phase III. In Phase I this included literature search, antenna array data acquisition and recording of data on the Keysort file cards. The analytical work of Phase II included complete card information review and other source review pertinent to element performance. This final report summarizes the completed effort and includes the latest results of the analytical study.

Documents were received from various sources such as DDC TABs and Indexes, Government Clearing House reports and documents from libraries. A constant review of these documents resulted in significant values and parameters recorded on the Keysort cards.

The summary of this report, is as follows:

Phase I: Documents received from DDC and other sources to date: 335

Total Keysort cards filled out with antenna element information: 403

Phase II: Analytical study of the complete investigation is finalized in this report.

TABLE 2-1

PROGRESS AND STATUS SUMMARY

PHASE	TASK	UNITS COMPLETE	STATUS
I Information Retrieval and Recording	Keysort File Card design and production	1000 cards received	Design complete-cards received and used
	Location and establishment of literature sources		335 Reports received from DDC and other sources.
	Literature information on cards	403 Cards filled out	
	Card notching	403 Cards	
	Element types, cards classified and grouped		403 Cards grouped by element types: slots, dipoles, etc.
II Analytical Study and Investigation	Literature study, element types and pattern from card information		Completed
	Extended analysis		Complete in this report
	Planning techniques		Keysort card file established. Review type discourse of analyzed data. Final presentation.
III Presentation of retrieved and analyzed information	Oral Presentation		Complete
	Tech. Doc. Rpt.		Complete
	Graphic, written presentation of retrieved literature information		Design complete - 403 cards with information recorded. 403 cards notched.

SECTION III

ANTENNA ANALYSIS

The following concludes the analytic investigation of the antenna element study under Contract No. AF 30(602)-3707. The recommendations and description of the study is summarized below.

Research showed large gaps apparent between experimental, theoretical studies of potential array element performance and conclusions actually reached for given element types to phased array antennas of medium or large magnitude. Conclusive facts primarily appeared in the linear current element areas, such as dipoles and certain spiral types. In the course of study, a partial or complete lack of theoretical visualization of phased array problems became apparent.

Project study showed that major effort had been expended to minimize mutual impedances, mutual couplings and such entities. This technique did not prove that a few elements minimized would be wholly eliminated for a large number. Decision was made to construct a theory of phased arrays and to test these theories with experimental research. The results of the research theory discussed in the previous reports are now combined with the final report.

The results presented inc ..

1. A method for the determination of the impedance properties of element arrays entirely from pattern measurements driving a single element.

2. For lossless antenna elements, a specific relationship between the power pattern of a single element in the array and the impedance properties, thereof.

3. A method of synthesizing certain classes of antenna elements in terms of their properties when immersed in an array rather than as isolated radiators.

4. A technique for handling infinite or very large arrays whose general simplicity and power recommends it as a tool for further investigation.

5. Suggested approaches for the theory developed for infinite arrays which promise to be fruitful for studying finite arrays.

The structure of steered arrays had to be examined to determine the applicable theory. A minimal result was obtained from tremendous expended effort accomplished on collections of arbitrary antennas in terms of their mutual impedances and matrices involving power integrals. General theories were applied equally to antenna array obtained from cleaning out antenna warehouses, antenna placement at random ground areas and study of their mutual impedances. It is evident that a highly organized structure is noted, when reference is made to steerable array antennas, particularly for radar application. Steerable antennas are composed principally of identical elements arranged in a regular pattern. The regular pattern consists of a group of motions, including reflections and rotations. These carry a portion of the antenna into another portion in the same sense that two triangles are congruent.

Consider then, infinite arrays so that various kinds of motion, carry the entire antenna into the entire antenna and the antenna structure is thus presented in its strongest light. In assuming the arbitrary elements rectangular grid, array functions consisting of a quantity or family of quantities must be obtained. The functions thus obtained from the values of a certain quantity defined at the various antenna elements as coefficients in a double Fourier series, prove to be a natural, strong method of treating array phenomena. Replace elaborate infinite matrix multiplications (and allied operations), with a simple multiplication scalar function of two variables. Evidence the fact that these functions are similar to ordinary impedance, etc. of circuit theory. Therefore ordinary circuit theory results suggest results for arrays.

Extended techniques in Section II are based on information outlined in Section I. As a result of this expanded theory, an essential description of the impedance properties of the array may be obtained, if array elements are terminated successively with three different impedances and the ratio of the radiated power densities under these three conditions for a unit power drive on one element is measured. The surprising result is that neither passivity nor reciprocity of the array under study, is required. If the antenna elements array is passive and lossless (not necessarily reciprocal), the pattern may be obtained. The measurements may involve ratio of quantities only. Measurement is such that the test antenna and antenna being measured need not be at a constant distance point-to-point, nor need the generator power out-

put remain constant any longer than is needed for the measurement at a given direction with the three different impedances. The particular impedance quantities derive from the way an antenna is used in practice. Drive combinations which produce suitable shape beams, side lobe levels, etc. in or over a given direction range are desirable rather than uniform impedance for arbitrary assignation of drives to the antenna element. Primarily, this means the combinations of drives which produce reasonable antenna beams not the apparent impedance properties for any drive combination. A function constructed by Fourier series technique from the reflection coefficient, self and mutual, of the array is shown to bear an intimate relationship for the beam formed by the antenna and an equal relationship to the powers reflected from the terminals of the antenna when driven to produce such a beam. The result is a desired antenna power angular density pattern to subsist over the angular region in which a scan is made of a pencil-like beam, if for the scan angles within that region the antenna terminals present an apparently constant impedance to the signal sources.

Previously, this result was derived for the lossless case and emphasis was made that it was not only undesirable for the mutual impedances, couplings, etc. to vanish but that it was impossible for them to vanish for rectangular spacings less than an amount appreciably greater than one-half wave length. The very constancy of apparently terminal impedances on all driven antenna terminals depends upon the mutual couplings, etc. not being zero.

In the first portion of this Section unlikely situations of impedance are discussed. The impedance properties depend on a series of mutual impedances multiplied by trigonometric terms to form a Fourier series. It is unlikely that a few terms of the series will represent the total sum adequately as the series is not absolutely convergent. In Section II, relationship between measured antenna patterns and impedance properties for lossless elements are defined in a more meaningful manner. In this case, the pattern properties of a single element determines the impedance properties of the array. To choose adequate elements for a steerable array antenna provided the elements produce suitable polarization patterns, the only criteria beyond physical convenience is that impedance presented to the transmitter to produce variety and scan of beam remain constant and that as a receiving array, the noise figure degradation attributable to the array is minimized. These are both wrapped up in the constancy of impedance seen by any terminal when the array beam is formed and scanned. The constancy of impedance experimentally determined by measurements on the power of a single element are discussed in previous chapters. To determine suitability of an element pattern, suggestions have been given as to Smith chart construction. Indication is that the pattern measurements produced by a single radiator element with a variety of at least three terminating impedances are of a greater value than mutual impedance patterns for one terminating impedance.

Recommendation is therefore made for an experimental array to be constructed for use on the RADC antenna range of between 11 X 11 and 13 X 13

with the facility for presentation of three distinct impedances to all the array elements. It should contain facilities for driving the center element only and measuring its pattern ratios for the three impedances. The antenna should be constructed on a reasonable grid spacing (changing the frequency will affect the variety) with standard connectors on the face. This would allow a variety of elements to be connected to it. Time consumed in measuring mutual and cross impedances have been the greatest cost expenditure on previous antenna array investigations. In addition to providing formulas, interest has been stimulated in other areas. For example, this analysis and study can be extended to more elaborate radiator systems.

Only a limited number of functions need to be calculated, the raw material is then provided for a large variety of antenna design. It is recommended that functions be calculated for a suitable range of dipole spacing. Minimum definitive answers should be obtained to the questions relating to one dipole in front of a ground plane, single dipole and parasitic radiators as a useful turnstile in front of a ground plane.

To better analyze the element patterns and to complete the study, the functional aspects of the previous report TR No. G-149-QR3, Section III have been included in this final report as Appendix I.

Appendix II consists of all publication key coding, Master Card Parts I and II, type of data and the source of information recorded.

APPENDIX I

1. Array Functions

We shall concern ourselves with plane antenna arrays constructed on a rectangular grid. All of the results derived have analogues for arrays on more general grids corresponding to the seventeen plane crystallographic groups. We restrict ourselves primarily in the interest of clarity.

The grid may be coordinatized by integer pairs in the usual fashion. Thus, if the directions along the grid axes are given names, say up and down and left and right, then the numbering of the grid points by integer pairs can be accomplished so that if (m, n) is the pair associated with a given point then $(m + 1, n)$ is the point immediately to the right of the first and $(m, n + 1)$ the point immediately above the first.

If we take the grid point $(0, 0)$ as the origin of a space coordinate system suitably oriented then the grid point (m, n) is at position $\left[l_1 m, l_2 n\right]$ where l_1 and l_2 are the spacings between adjacent elements along the two axes of our grid.

We now assume that an infinite array of identical antenna elements are placed on the grid with one element at each grid point. For the time being these elements may be quite complex and in particular may have many terminals each. The important thing is that the elements be identical and have identical orientation except for their particular grid point. Thus if the coordinate

systems were translated along either or [illegible] the origin were at a new grid point any description of the array in the first coordinate system would be equally valid in the second systems. We shall call this property translation invarience.

Now there are various quantities defined for each element at each grid point. For example, if each element has a terminal labeled 1 we might write the voltage on terminal 1 of the element at position (m, n) as $e_1(m, n)$. At some position on each element, identical relative to the grid point of the element, there may be a vector current density. We might label the position on the element α and write the vector density at α for the element at (m, n) as $\underline{i}_{\alpha}(m, n)$. In general we will reserve subscripts for labels with respect to an element and use grid coordinates as arguments.

Quantities such as voltages, currents, incident waves, etc. are defined for each element and are a function of the particular element grid location as the notation introduced above indicates. In general such electrical quantities depend upon the particular excitation applied. Other quantities are functions of the array structure. These include impedances, admittances, reflection coefficients, and other transfer quantities. Now since the elements are identical, and any one is in the same environment as any other, structural quantities which depend only on a single element are constant from element to element. Thus for example if $z_1(m,n)$ is the impedance at terminal 1 of element

[illegible] when all other terminals of all other elements are open circuited then $z_1(m, n) = z_1(p, q)$ where (p, q) is the position of some other element, and hence any other element. We can therefore write $z_1(m,n) = z_1$ with no arguments.

A most important class of quantities depend not upon one element but upon two or more. For example, the mutual impedance between terminal 1 on the element (m,n) and terminal 1 on element (p, q). We can write this as $z_{11}(m,n; p, q)$. The definition assumes that all other terminals are terminated with the same termination impedance for each equivalent terminal on each element. Now we come to the far reaching consequence of translation invarience. Since $z_{11}(m,n; p, q)$ depends only on the structure of the array translation invarience implies that

1.1 $$z_{11}(m, n ; p, q) = z_{11}(m + r, n + s; p + r, q + s)$$

for any integers r, s. We can thus write

1.2 $$z_{11}(m , n; p, q) = z_{11}(m-p, n-q).$$

The arguments in $z_{11}(r, s)$ as in the right hand side of 1.2 are <u>not</u> element positions or grid point coordinates. They do define the translation which carries the second of the elements, upon which the quantity depends, into the first upon which it depends. A similar argument can be applied to any quantity

whatsoever which depends upon two elements. For example, we might have $y_{12}(r, s)$ as the mutual admittance between terminal 1 on the first element and terminal 2 on the second. The pair (r, s) defines the translation which carries the second element into the first. In general then for structural quantities Q depending upon two elements,

$$Q(m, n; p, q) = Q(m - p, n - q) \tag{1.3}$$

Now for purely mathematical reasons it is a little inconvenient for some quantities to depend on element positions and others on translations. We observe however that the quantities which depend on position are just those which will arise when some set of generators, etc. are attached to the array. If the generators were disconnected and moved to a new location on the array and reconnected in the same relative fashion as before, all of the quantities dependent on position would appear as before except for a translation. The quantities thus depend upon position only by an implied reference to element $(0, 0)$ as the origin of the position coordinate system. That is to say, we may take position as defined by a translation from the origin. As a result we may regard the arguments in expressions like $e_1(m,n)$ as translations and $e_1(m,n)$ as a quantity depending upon element $(0, 0)$ thus

$$e_1(m,n) = e_1(m,n; 0, 0) \tag{1.4}$$

The manner in which it depends upon element (0, 0) is simple: the dependence is that (0, 0) is the origin of a coordinate system within which we will discuss some particular excitation or other array properties whose nature is not translation invarient.

The above may seem to be an unnecessary complication of thought however the success of the method to be employed depends upon this identification of the quantities considered as functions on the members of the translation group under which the array structure is invarient.

For simplicity suppose now that we have only one terminal at each location. Letting $e(m,n)$ be the voltage (at (m,n)), $i(m,n)$ be the current and $z(m,n;p,q)$ be the mutual impedance between (m,n) and (p,q) we have

$$1.5 \qquad e(m,n) = \sum_{p,q} z(m,n;p,q)\, i(p,q)$$

where the summation extends over all (p,q) for which $i(p,q) \neq 0$. By 1.2 this can be written

$$1.6 \qquad e(m,n) = \sum_{p,q} z(m-p,\ n-q)\, i(p,q)$$

Similarly with a mutual admittance $y(r,s)$ we have

1.7 $$i\,(m,n) = \sum_{p,\,q} y\,(m-p,\; n-q)\; e\,(p\,,\,q)$$

Since 1.6 and 1.7 describe the same structure and either the driving currents or the driving voltages can be chosen independently, we have

1.8 $$\delta\,(m-r, n-s) = \sum_{pq} z\,(m-p,\; n-q)\; y\,(p-r,\; q-s)$$

$$= \sum_{pq} y\,(m-p,\; n-q)\; z\,(p-r,\; q-s)$$

where

1.9 $$\delta\,(0\,,\,0) = 1$$

$$\delta(r, s) = 0 \quad \text{if } r \neq 0 \text{ or } s \neq 0 \text{ or both}$$

We remark that the existance of z and y as finite quantities for all positions is assumed. The summations in 1.8 are in general infinite. Further in general either 1.6 or 1.7 involves an infinite summation. Now suppose, by measurement or other means, we are given the mutual impedances $z\,(m,n)$ and we wish to calculate the currents resulting from a given set of drive voltages. First we must solve an infinite set of simultaneous equations in infinitely many variables using equation 1.8. Next we must take the $y\,(m,n)$ thus obtained and compute an infinite number of possible infinite sums using equation 1.7 for the $i\,(m,n)$. To do this directly is scarcely possible. We must find an indirect method. The

technique to be described turns out to have a significance beyond computational convenience.

Let us form the (formal) sum

$$1.10 \qquad e(x, y) = \sum_{p,q} e(p, q)\, e^{j\pi(px + qy)}$$

where out of deference to the electrical convention $j = \sqrt{-1}$. Thus $e(p, q)$ is the coefficient of $e^{j\pi(px + qy)}$ in the Fourier expansion of $e(x, y)$ in the interval $-1 \leq x \leq +1, -1 \leq y \leq +1$. Obviously, $e(x, y)$ is periodic in x and y with period 2 in each variable. In a similar fashion define

$$1.11 \qquad z(x, y) = \sum_{pq} z(p, q)\, e^{j\pi(px + qy)}$$

$$i(x, y) = \sum_{pq} i(p, q)\, e^{j\pi(px + qy)}$$

and consider the formal derivation

1.12 $$z(x,y)\, i(x,y) = \sum_{p,q} z(p,q)\, e^{2\pi i(px+qy)} \sum_{r,s} i(r,s)\, e^{2\pi i(rx+sy)}$$

$$= \sum_{p,q,r,s} z(p,q)\, i(r,s)\, e^{2\pi i[(p+r)x+(q+s)y]}$$

$$= \sum_{m,n,r,s} z(m-r,n-s)\, i(r,s)\, e^{2\pi i(mx+ny)}$$

$$= \sum_{m,n} e(m,n)\, e^{2\pi i(mx+ny)}$$

1.13 $$z(x, y)\ i(x, y) = e(x, y)$$

We could therefore carry out 1.6 by finding $z(x, y)$, $i(x, y)$ by 1.11, multiplying the results to find $e(x, y)$ and then finding $e(p, q)$ by the Fourier inversion formula.

1.14 $$e(p,q) = \frac{1}{4} \int_{-1}^{1}\int_{-1}^{1} e(x,y)\, e^{-2\pi i(px+qy)}\, dx\, dy$$

We would have two infinite summations and as many integrations as we desire values of e (p, q). This might be easier than many infinite summations but the advantage is not great. A more clear-cut advantage arises when y (p, q) is wanted or i (p, q) as a function of e(p, q). If we start with 1.8 and set m-r = a, n-s = b then since the range of summation is infinite

1.15 $$\delta(a, b) = \sum_{pq} z(a - p, b - q) \cdot y(p, q)$$

where we have substituted p-r $\rightarrow$ p, q-s $\rightarrow$ q. This is of the same form as 1.6. If we define y (x, y) as was z (x, y) in 1.11 and form the product z (x, y) y (x, y) then just as in 1.12

1.16 $$z(x, y)\, y(x, y) = \sum_{pq} \delta(p, q)\, e^{2\pi i(xp+yq)}$$

or since all but one term vanishes on the right

1.17 $$z(x, y)\, y(x, y) = 1$$

or

1.18 $$y(x, y) = \frac{1}{z(x, y)}$$

This could be deduced from 1.13 but the derivation for the multiplication of two infinite matrices involved in the passage from 1.8 to 1.15 to 1.16 is important in its own right. By a repetition of these steps we can show that if $P(m,n)$, $Q(m,n)$ and $R(m,n)$ are quantities (basicly infinite matrix elements) such that

$$1.19 \quad R(m-r, n-s) = \sum_{p,q} P(m-p, n-q)\ Q(p-r, q-s)$$

with obvious definitions for $P(x,y)$, $Q(x,y)$, $R(x,y)$

$$1.20 \quad R(x,y) = P(x,y)\ Q(x,y)$$

Our results are so far formal only. Clearly if all of the series involved in the definitions converge absolutely there is no problem of existence. This is too strong a requirement by far. For our purposes we observe that if $a(p,q)$ is a quantity such that

$$1.21 \qquad \sum_{pq} |a(p,q)|^2 < \infty$$

then $a(x,y)$ exists "almost everywhere". The sums in equations such as 1.10 to define the function of x, y may have to be replaced by summation processes. The theory of these for double Fourier series is complicated and will not add to our physical insight in the present context. It is possible to justify what

we are doing quite rigorously under quite non-restrictive assumptions, however, the mathematical techniques involved are not at all elementary and to a large extent not those used in engineering work normally. For the moment then we shall assume the existence of the functions involved beyond what can be obtained from 1.21.

We shall call the functions of the x, y coordinates we have thus introduced <u>array functions.</u> Thus if Q (m, n) is any quantity defined on the array we define the array function corresponding to it as a function Q (x, y) such that

$$1.22 \qquad Q(m,n) = \frac{1}{4}\int_{-1}^{+1}\int_{-1}^{+1} Q(x,y)\, e^{-2\pi(mx+ny)}\, dx\, dy$$

We use this reverse definition rather than the series 1.10 to avoid the summation questions mentioned above. For practical purposes the series form will suffice inasmuch as they yield the same result whenever the series converges and Q(x, y) exists. From the completeness properties of trigonometrical functions Q(x, y) is uniquely determined almost everywhere by 1.22.

We have in the course of the preceding introduced voltage, current, impedance and admittance array functions. We observe that the mutual impedance properties of the array collapse into one function $\mathcal{Z}(x, y)$. If we continue with our array with only one terminal per element it is natural to consider the terminals as terminated in some resistance r_o and consider the traveling wave representation of the conditions at the terminals. Using $\mathfrak{a}(p, q)$ for the incident

wave on the terminal (p, q) and b (p, q) for the wave reflected at the terminal (p, q) we have

1.23 $$i\ (p,\ q) = \frac{1}{\sqrt{r_0}}\ (a\ (p,\ q) - b\ (\ p,\ q))$$
$$e\ (p,\ q) = \sqrt{r_0}\ (a\ (p,\ q) + b\ (\ p,\ q))$$

or in terms of array functions

1.24 $$i\ (x,\ y) = \frac{1}{\sqrt{r_0}}\ (a(x,\ y) - b\ (\ x,\ y))$$
$$e\ (e,\ y) = \sqrt{r_0}\ (a\ (x,y) + b\ (x,y))$$

Now a and b are connecte.. by a scattering matrix of reflec..on coefficients (assuming that there are no signals arriving from the ou..side and the array is passive). That is, we have quantities s (p, q) such that

1.25 $$b\ (m,n) = \sum_{pq} S\ (m-p,\ n-q)\ a\ (p,q)$$

Now suppose only element (0, 0) is driven so that all other a (p, q) are zero. Let the drive on (0, 0) be unit incident power and zero phase so a (0, 0) = 1. In this case we have

1.26 $$a\ (x,y) = 1$$

Furthermore, we have

1.27 $$b\,(m,n) = S\,(m,n)$$

for this case only. Now the total power reflected back is

1.28 $$\begin{aligned} P_{res.} &= \sum_{m,n} |b(m,n)|^2 \\ &= \sum_{m,n} |S(m,n)|^2 \end{aligned}$$

and since the total incident power was unity there results

1.29 $$\begin{aligned} 1 &\geq P_{res.} \\ 1 &\geq \sum_{m,n} |S(m,n)|^2 \end{aligned}$$

and hence by condition 1.21 the S (x,y) array function exists. We can write our function equivalent of 1.25 as

1.30 $$b\,(x,y) = S\,(x,y)\; a\,(x,y)$$

and as long as we only have a finite number of a (p, q), non zero, b (x,y) will exist. In fact we need not restrict ourselves thusly. Any drive such that 1.21 is satisfied and hence of finite total power causes a (x,y) to exist and hence b (x,y). In this connection we mention that Parsivals Theorem for Fourier Series gives us

1.31 $$\sum_{pq} |a(p,q)|^2 = \sum_{pq} \bar{a}(p,q)\, a(p,q)$$

$$= \frac{1}{4} \int_{-1}^{+1} \int_{-1}^{+1} \bar{a}(x,y)\, a(x,y)\, dx\, dy$$

and thus we can compute the total power from the array functions. The total reflected power is then

1.32 $$P_{ref} = \frac{1}{4} \iint \bar{b}(x,y)\, b(x,y)\, dx\, dy$$

$$= \frac{1}{4} \iint \bar{S}(x,y)\, S(x,y)\, \bar{a}(x,y)\, a(x,y)\, dx\, dy$$

1.33 $$P_{ref} = \frac{1}{4} \iint |S(x,y)|^2 |a(x,y)|^2 dx\, dy$$

We may find the wave reflected from any given terminal using the inversion 1.32 and in short

1.34 $$b(p,q) = \frac{1}{4} \iint S(x,y)\, a(x,y)\, e^{-j\pi(px+qy)}\, dx\, dy$$

Equations 1.33 and 1.34 are very important.

Now we can obtain a simple physical insight into the meaning of the array function a (x, y). Suppose for a moment we consider an hypothetical array of isotropic radiators with half wave spacing on a square grid. If a (p, q) is taken

in this array to be the current in the (p, q) radiator we can compute a hypothetical far field pattern. If we let X be the cosine of the angle between the direction in which we are examining the field and the p axis of the array and y be the cosine of the angle between the direction and the q axis then the far field is given by

$$F(x,y) = \sum_{p,q} a(p, q)\, e^{+i\pi(px + qy)}$$

1.35 $$= a(x, y)$$

Thus for $x^2 + y^2 \leq 1$ we can interpret a (x, y) as a far field pattern obtained by applying the a (p, q) as drives on an array of isotropic radiators with half wave spacing. For $x^2 + y^2 > 1$ we are not so fortunate unless we allow ourselves to think of beams, etc. at imaginary angles. Now in isotropic array beam synthesis beams at imaginary angles occur and in general cause an apparent increase in the reactive power in the aperture (usually defined in terms of sums of squared currents). This is held to be undesirable. We shall be able to give more precision to this line of thought and indeed shall find that power from the hypothetical array scattered or radiated into these imaginary directions does have a deleterious effect on array performance for real elements.

We can interpret a (x, y) as a pattern obtained from a second hypothetical array of isotropic radiators, this time on the same grid as the original array. In this case let u be the direction cosine to the p axis and v be the cosine to the q axis, then the hypothetical field is

1.36 $$F(u,v) = \sum_{p,q} a(p,q)\, e^{i2\pi\left(\frac{l_1}{\lambda}pu + \frac{l_2}{\lambda}qv\right)}$$

where l_1, l_2 are the element spacings and λ is the wavelength. Again we see easily that

1.37 $$F(u,v) = a\left(\frac{2l_1 u}{\lambda}, \frac{2l_2 v}{\lambda}\right)$$

Now in this case since the spacing is usually greater than one-half wavelength we find grating lobes which arise since a (x, y) is periodic. a (x, y) thus becomes the pattern of an array of isotropic radiators with the same spacing as the original array but with scale factors supplied to the direction cosines. For reference we write explicitly

1.38 $$x = \frac{2l_1 u}{\lambda}, \quad y = \frac{2l_2 v}{\lambda}$$

$$u = \frac{\lambda x}{2l_1}, \quad v = \frac{\lambda y}{2l_2}$$

There is thus a quite natural correspondence between the spacial directions in front of the array and the array function variables. Also, for reference we give the transformation equations from spherical polar coordinates into the u, v system. Let ϕ be the angle between the radius vector and the normal to the plane of the array. Let θ be the azimuth angle measured from the first axis. Then

1.39 $$\cos \phi = \sqrt{1 - u^2 - v^2}$$

$$\tan \theta = \frac{v}{u}$$

and for the element of solid angle

1.40 $$\sin \phi \, d\phi \, d\theta = d\Omega = \frac{du\,dv}{\sqrt{1-u^2-v^2}}$$

or in a mixed system

1.41 $$d\Omega = \frac{du\,dv}{\cos \phi}$$

We shall also give the transformation from the spherical system into x, y coordinates. They are

1.42 $$x = \frac{2l_1}{\lambda} \cos \theta \sin \phi$$

$$y = \frac{2l_2}{\lambda} \sin \theta . \sin \phi$$

1.43 $$d\Omega = \frac{\lambda^2}{4l_1 l_2} \frac{dx\,dy}{\cos \phi}$$

1.44 $$\cos \phi = \sqrt{1 - \frac{\lambda^2}{4l_1^2} x^2 - \frac{\lambda^2}{4l_2^2} y^2}$$

Because of the close analogy between the array functions and patterns and between array function variables and space direction cosines we suspect an intimate connection between radiated patterns from the array with complicated elements and the array functions. We shall see that this is so

Before proceeding to specific analyses of array element problems, we wish to point out the fundamental property of the array functions which is that they can be manipulated exactly like scalar impedances. Thus with one terminal on the antenna element the relationships of change terminal impedance voltage current, etc. are just the same as for a single terminal except that the quantities are array functions. If there are N terminals on the element then equations arise which are those of N-terminal theory except that again instead of numbers we have array functions. Thus we would in the case of N terminal antenna elements deal with N vectors and N x N matrices whose terms are array functions. The commutativity induced on the infinite array matrices (which make the array function approach possible) does not apply to these finite matrices of course. In fact, one could write Maxwell's Equations in terms of array functions obtained from the field quantities. We shall find the germ of an element synthesis procedure herein.

The array functions would be functions of the frequency as well if finite bandwidths are considered. More generally they could be functions of time. In this case the analogues of impedances etc. become linear operators. Linear operators can be turned into array function operators. A treatment of transient

array behavior can be started from this basis.

Finally, the treatment of finite arrays can use array functions as a starting point. The technique involves integral equations which represents a complication; on the other hand, the structure of the array is built in, so to speak. An alternate approach to finite array questions is suggested by array function theory. This involves defining a sort of finite array function. Just as ultimately the array quantities were made to be functions of the elements of a translation group and the array functions obtained therefrom so for finite arrays the array quantities can be made functions of the elements of a group of transformations which leave the array structure invarient. An analogue of the array functions can then be constructed depending upon the theory of group characters. Whether or not this appraoch is fruitful is unknown.

In the preceding we have set up rather elaborate machinery for studying antenna array problems. In the next chapters we must show that it does in fact meet the promises set forth above.

2. Far Fields

In the preceding chapter we set up the theory of array functions for an infinite array of antennas. We did not, however, mention anything about the radiation field of the antenna elements themselves although we did interpret some of the incident wave array functions in terms of radiation patterns from hypothetical arrays of isotropic radiators. We shall start out with the barest minimum of information about far-fields. We shall define a far-field field quantity at a single frequency as one which

a. Is a function of the direction from the center of the array face.

b. If the array origin (including generators, etc) is translated by a translation (p, q) the quantity is multiplied by

$$e^{j2\pi\left(\frac{s_1}{\lambda}up + \frac{s_2}{\lambda}vq\right)}$$

c. Is a linear function of the incident waves, impressed voltages, etc. at the antenna terminals.

An example might be the voltage in a test dipole at a very great (infinite) distance from a fixed point on the array plane. For each direction from the fixed point the distance and orientation are prescribed but need not be constant.

By virtue of (c) if f is the quantity then for an incident wave on the terminal of element (p, q) then

2.1 $$f = f(p,q)\,a(p,q)$$

Holding a constant, by (b) we have

2.2 $$f(p+r, q+s)\,a = e^{j2\pi\left(\frac{\ell_1 u}{\lambda}r + \frac{\ell_2 v}{\lambda}s\right)} f(p,q)\,a$$

and hence we have the existence of $f_0(u,v)$ such that

2.3 $$f(p,q;u,v) = e^{j2\pi\left(\frac{\ell_1 u}{\lambda}p + \frac{\ell_2 v}{\lambda}q\right)} f_0(u,v)$$

For any arbitrary $a_{01}(p,q)$ drive assignment we have then for the field quantity produced

2.4 $$\begin{aligned} f_1(u,v) &= \sum_{p,q} f(p,q;u,v)\,a_{01}(p,q) \\ &= \sum_{pq} f_0(u,v)\,e^{j2\pi\left(\frac{\ell_1 u}{\lambda}p + \frac{\ell_2 v}{\lambda}q\right)} a_{01}(p,q) \\ &= f_0(u,v)\,A_{01}\left(\frac{a\ell_1}{\lambda}u, \frac{a\ell_2}{\lambda}v\right) \end{aligned}$$

where the quantities on the right of the last equation are array functions. We can introduce x and y as direction coordinates by using 1.38 and writing

2.5 $$\hat{f}_0(x,y) = f_0\left(\frac{\lambda x}{2 l_1}, \frac{\lambda y}{2 l_2}\right)$$

we have

2.6 $$\hat{f}_1(x,y) = \hat{f}_0(x,y)\, a_{01}(x,y)$$

Writing a superscript asterisk for complex conjugates (adjoint if the $\hat{f}$ are vector or tensor quantities) then defining

2.7 $$P_k(x,y) = \hat{f}_k^{*}(x,y)\, \hat{f}_k(x,y)$$

as the squared absolute value for $\hat{f}_k(x,y)$ we have

2.8 $$P_1(x,y) = P_0(x,y)\, a_1^{*}(x,y)\, a_1(x,y)$$

For example, if f were the voltage on a test antenna then P would be proportional to the received power on its terminals. If a_1, a_2 are two different drive assignments then we have for the ratio of the squared field quantities

$$2.9 \qquad R_{12}(x,y) = \frac{P_1(x,y)}{P_2(x,y)} = \left| \frac{a_1(x,y)}{a_2(x,y)} \right|^2$$

The ratio of the squared field quantities in a direction whose corresponding x and y are given by 1.38 is then proportional to the square of the absolute value of the ratio of the corresponding drive array functions. We remark that no assumptions as to passivity, reciprocity, etc. have been made about the array. We shall assume only one thing.

(d) Let the array be such that for any finite incident power at the terminals the reflected power is finite. (in practical terms it doesn't oscillate)

This is sufficient to guarantee the existence of a scattering array function S (x, y) by Chapter 1.

Now for a reciprocal lossless junction we have the scattering matrix for two terminals

2.10 $$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

where

$$S_{11} = \tanh \gamma \exp(-2j\alpha)$$

2.11 $$S_{22} = -\tanh \gamma \exp(-2j\beta)$$

$$S_{12} = S_{21} = \operatorname{sech} \gamma \exp(-j(\alpha+\beta))$$

representing in succession from terminals 1, a transmission line of length α (radians) at the reference impedance of terminals 1, a transformer of equivalent ratio $\exp(-\gamma)$ and a transmission line of length β at the reference impedance of terminals 2. If the reference impedance at terminals 2 is r_2 and the junction is such as to present at terminals 2 an impedance of r_1 then when terminals 1 are matched to their reference

2.12 $$\exp(-\gamma) = \sqrt{\frac{r_1}{r_2}}$$

2.13 $$\tanh \gamma = \frac{r_2 - r_1}{r_2 + r_1} = -S$$

where ρ is the reflection coefficient of r_1 with respect to r_2.
We can arrive at any desired source impedance at terminals 2 by inserting a transformer as above and placing a line of length, a suitable β at impedance r_2 between the transformer and the terminals 2. Therefore we may write, taking α as zero.

$$
\begin{aligned}
& S_{11} = -\rho \\
2.14 \qquad & S_{22} = \rho \exp(-2j\beta) \\
& S_{21} = S_{12} = \sqrt{1-\rho^2} \exp(-j\beta)
\end{aligned}
$$

where the sign of the root must agree with ρ.

Let us reassign the subscript 0 to the antenna terminals and let r_0 be the reference impedance for which a scattering function $S_0(x,y)$ is defined. Let us further assume that the terminals 2 of the junction have reference impedance $r_2 = r_0$ and are connected to the antenna terminals. At any one element then we have (ignoring element arguments)

$$
\begin{aligned}
2.15 \qquad & a_0 = b_2 \\
& b_0 = a_2
\end{aligned}
$$

the first of equations 2.10 can be written then

2.16 $$a_0 = S_{21}\, a_1 + S_{22}\, b_0$$

Now if this same junction is attached to all elements thus changing the termination of them all in like fashion we may pass to array functions and 2.16 yields

2.17 $$a_0(x,y) = S_{21}\, a_1(x,y) + S_{22}\, b_0(x,y)$$

but

2.18 $$b_0(x,y) = S_0(x,y)\, a_0(x,y)$$

and hence

2.19 $$a_0(x,y) = \frac{S_{21}}{1 - S_{22}\, S_0(x,y)}\, a_1(x,y)$$

Now if a single element (o, o) is driven through the junctions we have for corresponding drives at the antenna terminals a_{oi}

2.20
$$a_{01}(x,y) = \frac{S_{21}}{1 - S_{22}\, S(x,y)}$$

since $a_1(x, y) = 1$ in this case for unit drive. We can now use 2.6 to compute the field due to unit drive on the array as seen through the junctions and hence the array terminated with the new impedance with reflection coefficient $\mathcal{S}\, e^{2j\beta}$ with respect to the old.

the result is simply

2.21
$$\hat{f}_1(x,y) = \frac{S_{21}}{1 - S_{22}\, S(x,y)}\, \hat{f}_0(x,y)$$

However, we do not need to use this directly to compute the ratio of squared field quantities but may use 2.20 directly in 2.9 with obvious change of subscripts.

$$R_{01} = \left| \frac{a_0(x,y)}{a_{01}(x,y)} \right|^2, \quad a_0 = 1$$

2.22

$$= \left| \frac{1 - S_{22}\, S_0(x,y)}{S_{21}} \right|^2$$

or in terms of ρ, β and letting $S_o(x,y) = |S_o(x,y)| e^{j\varphi_o(x,y)}$ whence φ_o is the phase of S_o

$$R_{o1} = \frac{1 - 2\rho|S_o|\cos(\varphi_o - 2\beta) + \rho^2|S_o|^2}{1-\rho^2} \tag{2.23}$$

Suppose that three measurements with different driving source impedances of the squared field quantity are taken and two ratios are determined for points in the field corresponding to some range of (x, y). Then in general 2.23 will allow us to determine S (x,y) in this range of (x, y) for these impedances. Taking one of the source impedances as the reference (actually its real part if complex) we can compute the ρ and β for each of the other two. The two ratios (as functions of (x, y) in the range) yield two equations of form 2.23. These can be solved in general for φ_o and $|S_o|$ and hence we have

$$S_o(x,y) = |S_o(x,y)| e^{j\varphi_o(x,y)}$$

In the particular case where the reference impedance is r_o, the second r_1 and the third obtained from r_1 by inserting a one-eighth wave line between the antenna terminals and the termination of characteristic impedance r_o the explicit result is

2.24 $$|S_0|^2 = \frac{1-\rho^2}{\rho^2}\left[\left(\frac{R_{01}+R_{02}}{1-\rho^2} - \left(\frac{R_{01}-R_{02}}{2}\right)^2 - \frac{1}{(1-\rho^2)^2}\right)^{1/2} - \frac{R_{01}+R_{02}}{2}\right]$$

2.25 $$\cos\psi_0 = -\frac{1+\rho^2|S_0|^2-(1-\rho^2)R_{02}}{2\rho|S_0|}$$

2.26 $$\sin\psi_0 = \pm\frac{1+\rho^2|S_0|^2-(1-\rho^2)R_{02}}{2\rho|S_0|}$$

where subscript 1 refers to the resistive termination and 2 refers to the version with the eighth wave line. For a choice of source resistances in which taking the first as the reference, the second as a higher with reflection coefficient $\rho > 0$ and the third as lower with a reflection coefficient equal in numerical value to the first but of opposite sign. Simpler formulas result. These are,

2.27 $$|S_0|^2 = \frac{(1-\rho^2)}{\rho^2}\left[\frac{1}{2}(R_{01}+R_{02}) - \frac{1}{1-\rho^2}\right]$$

2.28 $$\cos\psi_0 = \frac{1}{2}\sqrt{1-\rho^2}\,\frac{\frac{1}{2}(R_{02}-R_{01})}{\sqrt{\frac{1}{2}(R_{01}+R_{02})-\frac{1}{1-\rho^2}}}$$

An ambiguity in the sign of $\frac{\varrho}{S_0}$ remains in this case. Subscript 1 refers to the source with the positive reflection coefficient.

We remind the reader that all of the quantities in 2.24 to 2.28 are functions of (x, y) in [illegible] ratios via the direction angles, the reflection quantities as array functions.

To complete this part of the picture we may use the first of equations 2.10 and obtain via 2.15

$$b_1 = S_{11} a_1 + S_{12} b_0 \tag{2.29}$$

and thence from 2.18 and 2.19

$$b_1 = \left(S_{11} + S_{12} S_0 (1 - S_{22} S_0)^{-1} S_{21} \right) a_1 \tag{2.30}$$

or using 2.14 and allowing for α, the line length from the transformer to the new input terminals.

$$b_1 = -e^{-j2\alpha} \frac{\varrho - e^{-j2\beta} S_0}{1 - e^{-j2\beta} \varrho S_0} a_1 \tag{2.31}$$

or in fine

$$2.32 \qquad S_1(x,y) = -e^{-j2\alpha}\,\frac{g - e^{-j2\beta}S_0(x,y)}{1 - e^{-j2\beta}g\,S_0(x,y)}$$

Since g, β, α can be found to lead to any source impedance whatsoever, we find that the scattering array function if known for one source impedance is known for all.

We may formulate our conclusions thus far as follows:

Let the ratios of the absolute values of a far field quantity be measured among the fields resulting from a fixed maximum available power signal applied to the terminals of an element in an infinite array for three distinct source impedances whose ratios are not all real. Let for each measurement all of the other element terminals be terminated with an impedance equal to the source impedance. Let these ratios be determined for a range of directions. Then the scattering array function s (x, y) can be computed for the (x, y) corresponding to angles within this range and for any source impedance.

Since the scattering array function determines the impedance properties of the array, the determination is of some importance. The interesting thing about the result is that no assumptions of passivity or reciprocity are made. The limitation of this method for a complete determination is that the range of (x, y) corresponding to all $u^2 + v^2 \leq 1$ may not span the required range $-1 \leq x \leq +1$, $-1 \leq y \leq +1$. It will, however, span the portion of the (x, y) square most involved in the impedance properties of beams. To see this, consider 1.33, namely

$$2.33 \quad P_{Ref.} = \frac{1}{4} \iint |S(x,y)|^2 |a(x,y)|^2 dx\,dy$$

and suppose a (x, y) are drives which produce a concentrated beam. By the discussion of Chapter 1 of the relationships between a (x, y) and the beams produced by arrays of isotropic radiators with a (x, y) as drives, and the relationships implied for a full pattern by 2.6 and 2.8, taking $\hat{f}$ as the full vector far field, namely multiplication of array power pattern by element power pattern, a (x, y) represents a concentrated beam in x, y space. Therefore, approximately

$$2.34 \quad P_{Ref} \approx \frac{1}{4} |S(x_0, y_0)|^2 \iint |a(x,y)|^2 dx\,dy$$

where x_0, y_0 correspond to the beam pointing angle. Noting that the incident power is (1.31)

$$2.35a \quad P_{inc} = \frac{1}{4} \iint |a(x,y)|^2 \, dx\, dy$$

we have

$$2.35\,b \quad P_{Ref.} \approx P_{inc} \, |S(x_0, y_0)|^2$$

In a real sense we can regard Pref as wasted power and hence we desire $|S(x_0, y_0)|^2$ to be small. Furthermore, if Pref is small enough then since it is total reflected power the individual terminals will be matched. We may also consider equation 1.34 which is

$$2.36 \quad b(p,q) = \frac{1}{4} \iint S(x,y)\, a(x,y)\, e^{-j\pi(px+qy)} \, dx\, dy$$

For elements such that the phase term in 2.36 is slowly varying and if the beam is good enough so that a (x, y) is significantly different from zero only in the neighborhood of X_0, Y_0, the pointing angle, then it is obviously desirable for s to be small or vanishing in the neighborhood of X_0, Y_0.

Now the X_0, Y_0 corresponding to beam pointing angles are in the region for which the measurement technique described above is effective. Thus the measurement would yield information as to the ability of the array to form beams such that the apparent driving point impedance at the terminals remains constant as the beam is steered. If the elements of the array are given, there is no particular reason why the particular impedances chosen for a measurement should include that for which the array terminals are matched relative to good beams. However, there may be an impedance which used as a source will give a suitably small s (x, y) in a scan region of interest. Since the changes in s (x, y) when the source impedance is changed is described by equation 2.32 and since all quantities on the right of this equation are constant except $s_o (x, y)$ the existence of a matching source impedance for a region R at (x, y) implies the constancy of S_o (x, y) over that region. Examination of the equations 2.24 to 2.28 then yields the requirement that the ratios be constant in that same region.

In more nearly real situations the constancy will not be perfect. Since 2.32 indicates that s (x, y) transforms as a scalar reflection coefficient, one can plot the results of the measurements on reflection coefficient paper (Smith

charts) and then find the impedance transformation to yield the best match for the scan region in question. There are convenient geometrical constructions for this.

The results above illustrate the power of the array function analysis in a most generalized case. The array can be active if desired and the above still hold. We are generally interested in array behavior before anyamplifiers are introduced and shall consider passive arrays in the sequel.

If we have a collection of radiators and field sources located entirely within a finite region of space, then subject to the radiation condition, viz. that if $\underline{\rho}$ is a radius vector from fixed point in space, r is its magnitude, $\underline{m}$ its unit direction vector so that $\underline{\rho} = r\underline{m}$ then the field vectors $\underline{E}$, $\underline{H}$ in suitable units satisfy

$$\underline{m} \times \underline{H} = -\underline{E} + o\left(\frac{1}{r}\right) \qquad r \longrightarrow \infty$$

2.37 $$\underline{E} = O\left(\frac{1}{r}\right) \qquad r \longrightarrow \infty$$

uniformly in all directions, then a far field exists. That is to say, there exists a vector field defined on the unit sphere such that

2.38 $$\underline{E} = \frac{e^{-j\omega r}}{r} \underline{f}(\underline{m}) + o\left(\frac{1}{r}\right)$$

$$\underline{m} \cdot \underline{f}(m) = 0$$

and from 2.37

2.39 $$\underline{H} = \frac{e^{-j\omega r}}{r} \underline{m} \times \underline{f}(\underline{m}) + o\left(\frac{1}{r}\right)$$

It follows that the angular density of power flow is then given by

2.40 $$P(\underline{m}) = \underline{f}^*(\underline{m}) \cdot \underline{f}(\underline{m})$$

and thus we can operate upon the unit sphere as far as integrated powers are concerned.

For infinite arrays, the existence of such a far field cannot be proved. The area in which this breakdown of proof occurs introduces the question of surface waves. Such waves can exist on a periodic structure. Now such surface waves fall into two classes: those which produce voltages or other reactions on the terminals of the elements, and those which do not. Those which produce no

reactions on antenna terminals can be ignored since then they cannot be excited by drives on the terminals. We can therefore restrict our considerations to those surface waves which present reactions to at least one set of terminals.

If all of the terminals are terminated with terminations with finite, non-zero, positive real parts, then no purely transverse surface waves can be propagated which possess reactions on the terminals. This can be deduced from conservation of energy considerations. For possible radial waves excited by an element, the situation is more complex. In general, the existence of such waves implies that combinations of elements can be excited so as to produce transverse waves not coupled to the element terminals of other elements which is impossible for transverse waves. For certain isolated frequencies, radial waves may be possible but not for a continuous range.

This gives a clue as to why the existence of the scattering array function was easy to prove with mild hypotheses since in general no surface waves are excited by elements when all are resistively terminated. Such is not necessarily the case with reactive terminations.

We then may assert that for finite resistive terminations the effective far field of an infinite array is the sum of far fields associated with each element and that the far field vector is a far field quantity as earlier defined.

Let now $\underline{f}_o(x, y)$ be this far field vector quantity. Then for any drive $a(x, y)$ we have for a complete field

2.41 $$\underline{F}(x, y) = \underline{f}_o(x, y)\, a(x, y)$$

The power density (angular) is given then by

2.42 $$\begin{aligned} P(x, y) &= \underline{F}^*(x, y) \cdot F(x, y) \\ &= \underline{f}_o^*(x, y) \cdot \underline{f}_o(x, y)\, a^*(x, y)\, a(x, y) \\ &= P_o(x, y)\, a^*(x, y)\, a(x, y) \end{aligned}$$

Now $P_o(x, y)$ is defined only for those x, y corresponding by the transformation equations 1.38 to real directions and furthermore is not periodic in x, y. We shall be concerned with total field power s given by

2.43 $$\begin{aligned} P_T &= \iint P_o(x, y)\, d\Omega \\ &= \iint P_o(x, y) \left(\frac{\lambda^2}{4 l_1 l_2}\right) \frac{dx\, dy}{\cos\phi} \end{aligned}$$

The range of integration is of course over just that set of x, y for which the cos ϕ as defined by 1.44 is real. We define a new power density function $\hat{P}_o(\psi, y)$ by

$$\hat{P}_o(\psi, y) = \frac{\lambda^2}{\ell_1 \ell_2 \cos\phi} P_o(\psi, y) : \cos\phi \text{ real}$$

2.44 $$\hat{P}_o(\psi, y) = 0 \quad \text{otherwise}$$

This is just the original power density multiplied by a factor depending on the array grid spacings and the angle of direction. The integration in 2.43 can be written

2.45 $$P_{oT} = \frac{1}{4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{P}_o(\psi, y)\, d\psi\, dy$$

Now all of our array functions are periodic and it would be convenient to have a field angular power density with similar behavior. We can obtain this by defining a grating lobe sum Σ' by

2.46 $$\Sigma' \hat{P}_o(\psi, y) = \sum_{r, s} \hat{P}_o(\psi + r, y + s)$$

where the sum on the right includes all non zero terms. It is easily seen to be periodic in x, y. The term grating lobe sum is natural. If we insert the x, y values in terms of u, v in the right hand side of 2.46 the sum becomes

$$2.47 \quad \sum_{r,s} \hat{P}_0 \left(\frac{2\ell_1}{\lambda}\left(u + \frac{\lambda}{2\ell_1} r\right), \frac{2\ell_2}{\lambda}\left(v + \frac{\lambda}{2\ell_2} s\right) \right)$$

Now for spacings sufficiently large and beam angles sufficiently large so that grating lobes appear, these lobes occur exactly at the u, v given by the terms in the sum 2.47. Hence, the name grating lobe sum. The grating lobe sum of a power density for a given direction is then the sum of the power densities in all of the directions for which beams would appear for an array of isotropic radiators on the same grid as the given array when the uniform beam is steered in the given direction. The integral in 2.45 can now be written

$$2.48 \quad P_0 T = \frac{1}{4} \int_{-1}^{+1}\int_{-1}^{+1} \sum{}' \hat{P}_0 (\psi, y)\, d\psi\, dy$$

For an arbitrary drive $a_1(x, y)$ we have

2.49
$$P_T = \frac{1}{4}\int_{-1}^{+1}\int_{-1}^{+1}\left\{\sum{}'\left(\hat{P}_0(x,y)\right)\right\} a^*(x,y)\, a(x,y)\, dx\, dy$$

where the a (x, y) terms can be factored out of the grating lobe sum since they are already periodic. Suppose now the array is passive and lossless. Then any power not reflected at the terminals must be radiated. Therefore

2.50 P incident - P reflected = P radiated

$= P_T$

and therefore from 1.31, 1.33 of Chapter 1

2.51 $\frac{1}{4}\iint\left(1-|S(x,y)|^2\right)|a(x,y)|^2\, dx\, dy = \frac{1}{4}\iint\left(\sum{}'\hat{P}_0(x,y)\right)|a(x,y)|^2\, dx\, dy$

We may pick a (x, y) arbitrarily and hence since 2.51 holds for all a (x, y) we have

2.52 $1-|S(x,y)|^2 = \sum{}'\, \hat{P}_0(x,y)$

Thus for the lossless case, the scattering array function magnitude can be obtained from the radiated power angular density. The determination is complete since by 2.44 $\hat{P}_o(x, y)$ is defined over the whole (x, y) space in terms of the values over that portion corresponding to real angles.

A number of conclusions were drawn from 2.52 in an earlier report. Most important was that for small s (x, y) outside of grating lobe regions the power pattern for one element had to approximate

$$P_o(u, v) = \frac{\ell_1 \ell_2}{\lambda^2} \cos\phi \tag{2.53}$$

This could be used as an element pattern criterion for measurement if absolute gains can be measured. It does not, however, tell the whole story from one pattern since the phase is not determined. If a second measurement of power pattern with a second terminating resistance is made, the single power ratio obtained would yield the phase by 2.25 since $|S_o|^2$ is already known from the absolute power measurement. The criterion is then the constancy of $|S_o|^2$ and ψ as before.

Now from 2.44, 2.46 and 2.52 we can infer that for a lossless array $|S_o|^2 = 1$ for all (x, y) not corresponding to real directions. ((x, y) is restricted to the rectangle $-1 \leq x \leq +1, -1 \leq y \leq +1$). For a grating lobe corresponding

to an imaginary direction of beam lies again in an imaginary direction. Thus for this case our previous determination in terms of power ratios can be completed as far as $|S_o|^2$ is concerned to include the whole x, y plane. As a result, our three pattern measurement allows for a complete determination of the total reflected power properties of the array for arbitrary drives by using 2.33. Furthermore, bounds can be placed on the unknown part of the waves reflected at any terminal by 2.36 since $|S|^2 = 1$ and hence $|S| = 1$ in the region for which the phase is unknown. The fact that in order that this unknown part be small, the isotropic pattern must be small in the unknown directions shows that excessive power in imaginary directions for the hypothetical array may adversely affect the impedance properties of the real array as stated in Chapter 1.

3. Some Element Considerations - Synthesis.

In Chapter Two we showed how for infinite arrays of lossless elements the impedance properties were essentially functions of the element patterns in the array measured at several terminating impedances. We required in general patterns at more than one impedance in order to establish the phase of the scattering array function. The traditional approach to these problems has been to measure patterns for one impedance and to measure the mutual impedances or couplings between elements. Furthermore, almost never are the patterns absolute so that at the best only the constancy or lack thereof of the magnitude of the scattering function for one impedance can be deduced.

The attempts to characterize large or infinite array behavior on the basis of measurements on small arrays of mutual impedances are doomed from the start to failure. Supposing that all patterns and mutuals in the small array were just exactly those of the large array the measurement of any reasonable number of mutual impedances or couplings is not sufficient to determine any of the array functions correctly. The various array functions arise as Fourier series with the coupling quantities as coefficients. They are not in general absolutely convergent series. The magnitudes of the coupling quantities (impedances, admittances or reflection coefficients) drop off about as the inverse square of the distance between elements on a ground plane. Now unless they drop off faster than inverse square uniformly in all directions except for some finite number the series do not converge absolutely. In this case, the approximation

of the series by a finite number of terms may be poor no matter how small the individual neglected terms are. One could check the approximation as far as magnitude is concerned by a measured pattern; however, the phase question is still vexatious.

The situation is bad enough when mutual reflection coefficients are measured. It is worse when mutual impedances are involved. The scattering function must converge a. e., the impedance array function may become infinite. It does this simply wherever s (x, y) = 1.

$$3.1 \quad Z(x,y) = r_0 \frac{1+s(x,y)}{1-s(x,y)}$$

Furthermore, s (x, y) is at least integrable square, $Z(x,y)$ need not be.

We shall find that it is possible to make statements about s (x, y) for finite samples of infinite arrays; no such statements seem to appear for $Z(x,y)$.
The impedance function has its best use in connection with elements for which the inverse distance and inverse distances squared dependencies can be obtained theoretically. Then for a large enough distance the remainder of the series is absolutely convergent and one may use measurements on smaller arrays to obtain estimates of the higher order terms and the near neighbor contributions.

We can use the array function method to synthesize elements of certain kinds which have optimum impedance properties for given scan domains. To see how this may be done, suppose that we have an array of elements with two terminals each. We desire to terminate the second terminal with a reactance so that the resulting element has the best impedance properties.

For two terminals per element we have in terms of array functions, suppressing the (x, y) arguments

$$b_1 = S_{11} a_1 + S_{12} a_2$$

3.2 $$b_2 = S_{21} a_1 + S_{22} a_2$$

we remark here that if reciprocity is assumed, we have

3.3 $$S_{21}(-x, -y) = S_{12}(x, y)$$

3.4 $$S_{11}(-x, -y) = S_{11}(x, y)$$

3.5 $$S_{22}(-x, -y) = S_{22}(x, y)$$

Equation 3.3 represents the way in which our array functions differ from scalar reflection coefficients. From 3.2 if we terminate terminal two in a reactance of reflection coefficient $\rho = e^{j\theta}$ then

3.6 $$a_2 = e^{j\theta} b_2$$

and hence from the second of 3.2

3.7 $$a_2 = (e^{j\theta} - S_{22})^{-1} S_{21} a_1$$

and finally from the first of 3.2

3.8 $$b_1 = (S_{11} + S_{21} S_{12} (e^{j\theta} - S_{22})^{-1}) a_1$$

We therefore have for the scattering array funcsion of the single terminal element thus obtained

3.9 $$S_\theta = (S_{11} + S_{21} S_{12} (e^{j\theta} - S_{22})^{-1})$$

If S_{11}, S_{21} and S_{22} are known θ can be picked to optimize S_θ by whatever criterion is to be employed. It is useful to express this process in

terms of the impedance array functions. If these are given, then

3.10 $$e_1 = Z_{11} i_1 + Z_{12} i_2$$
$$e_2 = Z_{21} i_1 + Z_{22} i_2$$

and for a reactance Xj at e_2 then as before

3.11 $$e_1 = \left(Z_{11} - (Xj + Z_{22})^{-1} Z_{21} Z_{12} \right) i_1$$

Now from the relation between the scattering array function and the impedance array function

3.12 $$S = \frac{Z - r_0}{Z + r_0}$$

we can compute the scattering function and hence both the impedance properties and the power patterns are immediate. We can regard the bracketed term containing Z_{22} as a new Z_{22} made up of the old with Xj in series with each. From this point of view we can regard Z_{11}, Z_{12}, Z_{22} as functions of parameters which include series impedances but which also might include positions, etc.

A practical application of this would be the synthesis of dipole arrays. For many practical purposes we can regard the effect of small changes in the length of a

dipole as that of placing a reactance in series with its center. Suppose now we calculated the self-impedance function of a grid of dipoles and the mutual impedance function between two grids, one in front of the other as a function of the additional parameter of spacing.

Let these functions be Z_S for the self-impedance function and $Z_M(d)$ fc the mutual function. First we may introduce a ground plane by representing the antenna as two planes of dipoles with a spacing equal to $2c$ where c is the dipole ground-plane spacing. The dipoles are driven out of phase in this representation then if e, i refer to the foward plane and $-e, -i$ to the rear.The equivalent of 3.10 is

$$
3.13 \quad \begin{aligned} e_1 &= Z_s i_1 - Z_m i_1 \\ -e_1 &= -Z_m i_1 + Z_s i_1 \end{aligned}
$$

and hence the impedance function is simply

$$
3.14 \quad Z(x, y; 2c) = Z_s(x, y) - Z_m(x, y; 2c)
$$

From 3.12 we could evaluate an S(x, y; d) and from this the powe. pattern and impedance/scan behavior. It should be possible in particular to optimize the dipole/ground-plane spacing fc a given range of scan angles.

Suppose now that we consider that there is a parasitic dipole also present with distance C_2 from the ground plane. In terms of their voltages and currents

$$e_1 = \left(Z_s - Z_m(2c)\right) i_1 + \left(Z_m(x(c-c_2)) - Z_m(x(c+c_2))\right) i_2$$

3.15 $$0 = \left(Z_m(x(c-c_2) - Z_m(x(c+c_2))\right) i_1 + \left(Z_s + Xj - Z_m(2c_2)\right) i_2$$

where X2 is the equivalent reactance for change of length.

For the impedance function of the combined antenna

3.16 $$Z = \left(Z_s - Z_m(2c)\right) - \frac{\left[Z_m(c-c_2 - Z_m(c+c_2)\right]^2}{Z_s - Z_m(2c_2) + Xj}$$

We can thus investigate the impedance and pattern properties as a function of spacings and lengths using 3.16. A useful technique would be to plot for a set of (x, y) in the scan range on a Smith chart for various values of C, C_2 and X2. There is no particular difficulty in the extension to N dipoles.

Implicitly the variation of grid spacings has been ignored. In practice the Z_s, Z_m are also functions of these. This technique also extends to other types of linear radiators.

It would seem worth while to compute the Z_m for a range of grid spacings and separations and the Z_s for the same range of separations. Rational (Hastings type) approximates should then be found so that fast computer subroutines for their calculation are at hand. It would then be feasible to compute optimum dipole arrays for various numbers of parasitic elements with respect to impedance properties or some specific pattern properties. These elements would undoubtedly behave better in large arrays than elements designed to behave well isolated, then inserted in an array with a hope that the good behavior will continue.

The utility of the array function analysis lies first in requiring only a limited number of functions to determine a variety of arrays and second in the fact that the synthesis is carried out in the array itself and the question of isolated versus array behavior does not arise.

APPENDIX II

ALPHABETICAL DIGITS FOR NOTCHING

AUTHORS' NAMES

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Mac or Mc	27
La, Le or L'	28
Du, De or D'	29

Hyphenated Names are keyed by first letter of first part followed by first letter of second part.

MASTER CARD

PART I

INFORMATION	LOCATION OF KEY
AUTHOR'S NAME	
First Letter of Last Name:	33 to 38; by digit number of the alphabet, using 33 to 36 as last digit, 37 as 10 and 38 as 20.
Second Letter of Last Name:	L1 to L6 as above.
JOURNAL CODE	
Title by Assigned Number (See Index of Notching Codes):	5 to 16 (Inclusive)
Volume Number (01 through 99):	17 to 24 "
Issue Number (01 through 99):	25 to 32 "
DATE PUBLISHED	
Last Figure (Year):	L9 to L12 (Inclusive)
3rd Figure (Decade):	L13 to L16 "
KEY by SINGLE NOTCH (if subject is discussed):	
Single Element:	R1
Array:	R2
Element Spacing:	R3
Scan Limitation:	R4
TYPE of POLARIZATION	
Horizontal:	R8
Vertical:	R7
Circular:	R6
Discussed But Not Specified; or Type Other Than Those Above:	R5

TYPE of ELEMENTS DISCUSSED	R9 to R12 (Inclusive)
ELEMENTS	**KEY**
Dipoles:	1.
Slots:	2.
Spirals:	3.
Helices:	4.
Log Periodic:	5.
Circular Horns:	6.
Rectangular Horns:	7.
Other Types:	8.
Combinations of Element Types:	9.
Loops:	10.

SECURITY CLASSIFICATION	1 to 4 (Inclusive)
Unclassified:	1.
Confidential:	2.
Secret:	3.
	4.
	5.

MASTER CARD

PART II

INFORMATION	LOCATION
NO. OF ELEMENTS	R13 through R16
Up to, and including 9:	1.
10 through 49:	2.
50 through 99:	3.
100 through 499:	4.
500 through 999:	5.
1000 through 2499:	6.
2500 through 4999:	7.
5000 through 9999:	8.
10,000 and more:	9.
discussed but no values provided:	10.
	11.
	12.
	13.
	14.
FREQUENCY RANGE	R17 through R20
Up to 299 MHz (inclusive)	1.
300 to 2999 MHz - UHF	2.
3000 MHz to 3.9 GHz - S Band:	3.
4.0 GHz to 8.1 GHz - C Band:	4.
8.2 GHz to 12.4 GHz - X Band:	5.
12.5 GHz and over:	6.
	7,
Discussed; but no Values Provided	8.
	9.
	10.
	11.
	12.
	13.
	14.

GAIN	L17 through L20
0 - 2.99 db:	1.
3 - 5.99 db:	2.
6 - 9.99 db:	3.
10 - 19.99 db:	4.
20 - 29.99 db:	5.
30 db and greater	6.
Theoretical:	7.
Discussed but no values provided:	8.
	9.
	10.
	11.
	12.
	13.
	14.

POWER HANDLING CAPABILITY	L21 through L24
Receivers:	1.
Less Than 25 W:	2.
25 W to 49 W:	3.
50 W to 99 W:	4.
100 W to 499 W:	5.
500 W to 999 W:	6.
1KW to 4.9 KW:	7.
5KW to 9.9 KW:	8.
10KW to 19.9 KW:	9.
20KW and greater:	10.
Discussed but no values provided:	11.
	12.
	13.
	14.

SIGNAL BANDWIDTH	R21 through R24
Narrow Band:	1.
Under 5%:	2.
5% to 7.49%:	3.
7.5% to 10%:	4.
Greater than 10%:	5.
Theoretical:	6.
Discussed but no values provided:	7.
	8.
	9.
	10.
	11.
	12.
	13.
	14.

BEAM WDITH	R25 through R28
Less than 0.5°	1.
0.5° to 0.9°	2.
1.0° to 2.0°	3.
2.0° to 4.9°	4.
5.0° to 9.9°	5.
10.0° to 19.9°	6.
20.0° to 49.9°	7.
50.0° to 100.0°	8.
Over 100°	9.
Theoretical:	10.
Discussed but no values provided:	11.
	12.
	13.
	14.

TERMINAL IMPEDANCE	L25 through L29
Less than 25 Ohm:	1.
25 to 49:	2.
50 to 99:	3.
100 to 199:	4.
200 to 399:	5.
400 to 599:	6.
600 to 799:	7.
800 to 999:	8.
1000 Ohm and greater:	9.
Input Impedance:	10.
Impedance Variation:	11.
Discussed; but no values provided	12.
	13.
	14.

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13. ABSTRACT

A final report of the investigation and analytical study of antenna array elements, determining the specific characteristics of array element performance for the designer, establishes a capability to select the optimum element. The literature survey is included with its key to the retrieval file. The literature and source data entered on the Keysort file cards is delineated within the accumulated data file.

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